International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN(P): 2249-6890; ISSN(E): 2249-8001
Vol. 8, Issue 3, Jun 2018, 1085-1096

INVESTIGATION ON TORSIONAL VIBRATION OF DRILL STRING IN CYLINDRICAL CAVITIES OF VERTICAL BORE-HOLE WITH LIQUID MEDIUM

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ABSTRACT

The research work deals with the torsional vibration of a drill string in a vertical cylindrical cavity of a bore-hole with liquid medium. The mechanical interaction models of the drill string with viscous liquid are investigated, Also this paper describes the vibrations of the drill string bit with allowance made for viscous friction, the nonlinear deferential equation with partial derivatives is used. The oscillation scheme of torsional auto-vibration of homogeneous drill string in the form of oscillation pendulum is stated. It was found that the properties of the liquid medium in which the rotating column, lead to a small range of angular velocity value, which are generated during the self-oscillation.

KEYWORDS: Vertical Bore-Holes, Auto-Vibration, Stationary Oscillations & Rotational Motions Dynamic Viscosity

Received: Apr 12, 2018; **Accepted:** May 03, 2018; **Published:** Jun 19, 2018; **Paper Id.:** IJMPERDJUN2018113

Original Article

1. INTRODUCTION

The problem of deep drilling vibration, buckling, bending and instability of drill string inside the cylindrical cavity of bore-hole attracts attention of many researcher, the whirling vibrations of rotating elastic drill string with ellipsoidal rigid bit in vertical cylindrical cavities was discussed by N. W. Musa, et al. 2015, he showed, that the ellipsoidal bit can change the direction of circumferential motion and stop its vibrations. The computer simulation and the numerical solution for drill string buckling inside cylindrical cavities of inclined bore-hole are studied too, the critical loads and the stability loss modes are founded. Based on the theory of flexible rods, the problem about elastic bending deformation of drill string in cylindrical cavities of curve bore-hole is discussed too, the external distributed forces, and their moments during the drill string trajectories motion in the second order equations are solved [1, 2, 3].

Drilling rig for drilling oil and gas wells in general includes: a drilling structure (derrick); round-trip equipment (winch); power equipment for driving the winch, rotor and drilling pumps, equipment for rotation of the drill string (rotary table); washing fluid that circulates in the cavity wells, and chisel (Figure 1).

One of the dynamic phenomena that contribute to the occurrence of emergency situation during drilling is the self-excitation of torsional vibrations of a rotating drill string (DS). Since DS is a torsion pendulum in the lower part of which, due to the dissipative interaction between the bit and the destructible rock is the outflow of energy from the drive mechanism to the environment, in violation of the outflow of the string can switch from the stationary equilibrium state of rotation in the torsion vibration mode. In this work, the task of computer modeling of these self-excited oscillations [4].

The reason for self-excitation of torsional vibrations in drilling rigs is bifurcation disruption of the balance of moments of elastic forces in the string and non-linear friction forces between the bit and the borehole wall [5, 6]. Parameter governing their stationary and self-oscillatory regimes is the angular velocity ω string rotation. With regard to the phenomena accompanying the rotation of DS, the investigation of the possibility of their generating self-oscillation enable us to answer three important questions: under what values of parameters of its functioning generation of their oscillation is possible; what type of regime self - exciting oscillations (soft or hard) occurs; what measures can pre-empt the possible modes of torsional oscillations [14-19].

For DS in comparatively shallow wells the answers to these questions can be obtained by using a simplified mathematical model of the oscillator with one degree of freedom based on a rotating torsion pendulum to the flywheel-bit which is applied to the nonlinear friction force of its frictional engagement with the destroyed breed [13].

However, if the length of DS is not small, the application of the model of the torsion oscillation of the pendulum is not justified for the analysis of its dynamics, since the oscillations of its elements cease to be co-phased and their modeling should be performed based on the wave differential equation. The necessity of such theory is indicated in the works [6, 14, 18, 19]. It is important to take into account the viscous friction forces acting from the side of the washing liquid to the string along its entire length.

2. STATEMENT OF PROBLEM ABOUT TORSIONAL VIBRATION OF DRILL STRING

In real conditions more complex forms of movements of the flywheel to a significant extent may contribute to the sticking effect of oscillation which is typical for systems with dry friction. It is in the short term stops the movement of the flywheel in the time intervals in which the sum of all moments of active forces and moments of inertia forces is less than some threshold moment of friction forces that must be overcome to make the flywheel spin

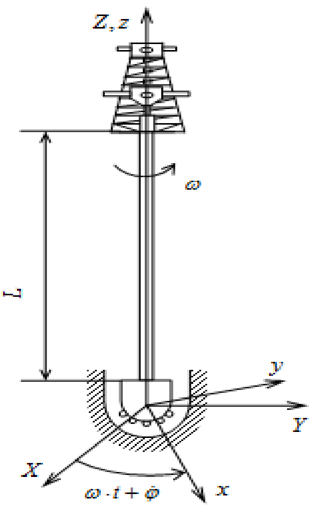

Figure 1: Structural Diagram of Drilling Rig

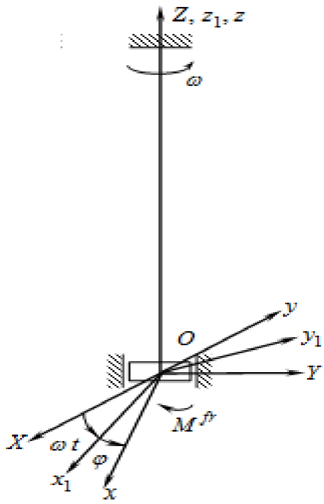

Figure 2: Design Scheme for Drill String

During these periods of drive installation on the upper end DS continues to rotate with an angular velocity ω, DS twists and potential energy of elastic deformations accumulates. After reaching the elastic moment at the DS values equal to the threshold moment of friction forces, the lower hand wheel (bit) starts to rotate, DS unwound and its potential energy begins to transform into kinetic energy of rotation of the string and flywheel. This rotation continues as long as the sum of elastic torque in the DS and the moment of inertia forces of the bit become smaller than the threshold values of friction forces, causing the flywheel stops again, etc. A characteristic of such periodic movements is that they occur with different speeds in forward and reverse movements and are accompanied by the occurrence of large accelerations, leading to the occurrence of supernumerary situations.

In the analysis of self-oscillations of the bit a significant impact on the shape of its movements can exert a force viscous force interaction between the pipe drill string and the wash fluid. It is known that this fluid like many other clay slurries and pastes refers to rheological applies to environments with non-Newtonian properties [4, 8, 9, 11, 12]. Therefore, their properties should be taken into account in the formulation of the considered problem.

The purpose of this paper is to develop the mathematical model describing the stationary oscillations of the drill strings in the cavities of the wells filled with wash fluid.

Consider the case of steady rotation the upper end of DS with a constant angular velocity ω. We introduce inertial coordinate system $OXYZ$ with beginning in the center of mass of the bit, the axis OZ coincides with the center line of DS (Figure 2). Regarding it with a speed ω rotates the coordinate system $Ox_1y_1z_1$. Tie with a chisel coordinate system $Oxyz$ rotating together with it, the axis Oz coincides with the axis OZ. Then the angle of rotation of the bit relative to system $OXYZ$ is $\omega t + \dot{\varphi}$ where ωt – the angle of rotation of the top end of the DS and the coordinate system $Ox_1y_1z_1$, t – time, $\varphi(z,t)$– angle of elastic torsion of DS, and $\varphi(0,t)$ angle of the elastic twisting of the chisel.

3. CONSTITUTIVE EQUATIONS OF DS VIBRATION INSIDE VERTICAL BORE-HOLE CAVITY

The dynamics of torsional vibrations of DS is necessary to study on the basis of the equation

$$\rho I_z \frac{\partial^2 \varphi}{\partial t^2} + f \frac{\partial \varphi}{\partial t} - GI_z \partial^2 \varphi / \partial z^2 = 0. \quad (1)$$

where ρ is the density of the material DS; I_z is the moment of inertia of area of the transverse cross-section; G is the modulus of elasticity of the material shear; k − coefficient which characterizes the moment of forces of viscous friction between the wash fluid and the outer wall of the pipe DS.

As stated in the paperwork [10], clay and cement mortars used at oil and gas enterprises for flushing of the well have properties of non-Newtonian fluids. Therefore, the coefficient k must be calculated using the values of the shear stresses in Couette flow between two cylindrical surfaces. The viscosity of the washing fluid with particles of crushed rock as for any dispersed system depends on these main factors:

- concentration of the dispersed phase;
- viscosity of the liquid phase;
- size and configuration of the particles;
- aggregation of particles;
- dissolved in a liquid medium macromolecular substances;
- content of emulsifiers and surfactants.

The rheology defines so-called Newtonian liquids, characterized by the fact that by the constant temperature the viscosity remains constant regardless of the shear rate at which the viscosity is measured [7]. For them, the tangential shear stress τ is determined by the dynamic viscosity coefficient μ and shift speed $\dot{\varepsilon} = \partial u / \partial y$ by formula

$$\tau = \mu \cdot \dot{\varepsilon} = \mu \partial u / dy \quad (2)$$

In Newtonian flow of liquid media, shear rate is always directly proportional to the tangential component of shear stress. In nature, a huge amount of liquid does not obey Newton's law of fluid flow, as their viscosity is dependent on shear rate (polymer solutions, suspensions, emulsions, oil). These types of liquids are classified as non-Newtonian, for which the connection between the shear rate and shear stress is described by complex nonlinear dependencies.

As a consequence of this interaction of the particles, a non-Newtonian fluid has a complicated structure in varying degrees depending on the structured nature of the interaction of components.

There are several types of non-Newtonian fluids. In the applied researches widely spread are models of plastic fluid (liquid or solid Bingham). In these kinds of fluids it is needed to make some initial effort to begin their flow, after which the dependence in coordinates shear stress - shear rate becomes straightforward. The viscosity of fluids at low shear rates is very large, and decreases rapidly with increasing this parameter, and is characterized by two constants, namely,

plastic viscosity and critical shear stress. An example of such systems is a solid plastic, for example oil, characterized by shear flow only when there is a voltage that exceeds the limit fluidity τ_0.

Satisfied with the simplest case of a plane shear rectilinear motion along the axis Ox with shear $\dot{\varepsilon} = \partial u / \partial y$, the rheological equation of such a plastic fluid flow can be provided in the form of:

$$\tau = \tau_0 + \mu' \dot{\varepsilon} \text{ when } \tau > \tau_0 \tag{3}$$

where τ_0 - limiting shear stress, μ' - structural dynamic viscosity (dot over the letter - the time derivative). When $\tau < \tau_0$, fluidity is absent, in other words, the medium behaves like a solid body.

Essentially nonlinear properties are present in pseudo plastic fluid, in which the viscosity shear rate changes accordingly, and any change in the viscosity characterizes the so-called viscosity, which is given for a given shear rate only. The viscosity of the pseudo plastic fluid is high at low shear rates and decreases with increasing shear rate. By these properties characterized are rubber and plastic materials comprising an anisotropic asymmetrical components, the interaction between them is weakened with increasing shear rate.

Pseudo-plastic fluids devoid of limiting the voltage stress, but their reduced viscosity determined by the coefficient that depends on the shear rate. Such "non-linear" liquids (suspension asymmetric particles, solutions of high polymers) are subject to the rheological equations of the type (Ostwald, Rayner)

$$\tau = k \dot{\varepsilon}^n, \tag{4}$$

where k and $n < 1$ are nearly constant over a wide range of voltages and

strain rates and reduced coefficient of viscosity $\tau / \dot{\varepsilon} = k \varepsilon^{n-1}$ decreases by increase of $\dot{\varepsilon}$.

The absence of a limit voltage brings pseudo plastic fluid to the so-called "dilatant" fluid, which, in contrast to the pseudo plastic, the reduced viscosity increases with increasing voltage ($n > 1$). This pattern is typical for suspensions of solid particles with their high concentrations, as well as starch pastes, which can not be attributed to concentrated suspensions of solid particles.

Choice of laws for shear stresses in the form of (3), (4) when $n < 1$ and $n > 1$ significantly complicates the equation of torsional vibration (1). However, given that fluctuations and DS bit occur in the vicinity of the status of their simple rotation with angular velocity ω, where the rotational motion of the fluid is close to the movement of Couette flow between two rotating cylinders, the equation (1) can be simplified. For that, it is sufficient to linearize this equation in a considered state of rotation speed ω consider the relation (2) for a Newtonian fluid, however μ viscosity coefficient in the formula (2) is calculated at a chosen value ω. In this formulation of the equation formula (1) will become linear:

$$\rho I_z \partial^2 \varphi / \partial t^2 + k \partial \varphi / \partial t - G I_z \partial^2 \varphi / \partial z^2 = 0, \tag{5}$$

However coefficient k in the equation is selected depending on the chosen speed ω.

It is determined by calculating hydrodynamic torque m^{fr} of distributed frictional forces during rotation of the inner cylinder into the outer cylinder chamber filled with liquid.

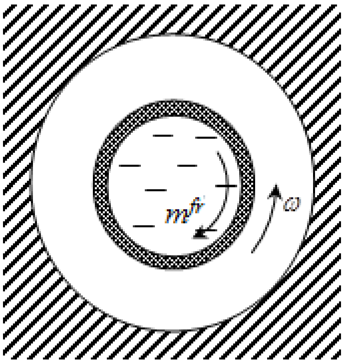

Figure 3: Calculation of the Viscous Coefficient of Friction and Torque m^{fr} with Internal Fluid Flow

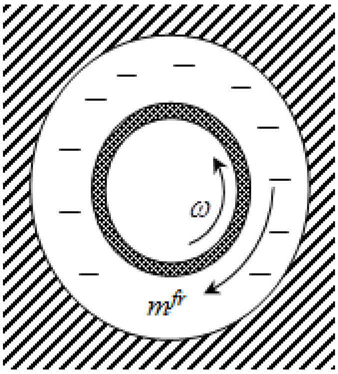

Figure 4: Calculation of the Viscous Coefficient of Friction and Moment m^{fr} during the Fluid Flow

A distinguishing feature of the drill string flow the washing liquid is the complex nature of its movement within the particles (Figure. 3) and the outside (Figure 4) of the drill pipe string. As the fluid flows in the axial direction between the rotating cylindrical surface of the column tube and a fixed cylindrical surface of the well, it can be approximately assumed that its elements move along helical paths. The problem of calculating the trajectories of these independent problem fluid mechanics to be solved separately for each process mode, different values of the axial velocity and rotational motions, the diameters of the cylindrical surfaces and mechanical properties of the fluid itself. Therefore, in this work we investigate the general patterns of the phenomena under consideration in a wide range of viscosity μ. In this case, however, for the evaluation the lower limit values of the coefficient μ, consider can be the case of the absence of axial motion of the liquid, shown in Figure. 4, with laminar flow of a Newtonian fluid.

Let the smaller and larger radii of the cylinders constitute r_1, r_2, respectively, and the smaller cylinder rotates with an angular velocity ω_1.

Then the function $v(r)$ of change of circumferential velocity along the radius r between the cylinders, provided stationary flow, is calculated according to the formula

$$v(r) = \frac{\omega_1}{r_2^2 - r_1^2}\left(\frac{r_1^2 r_2^2}{r} + r_1^2 r\right) \tag{6}$$

With its help we calculate the torque forces of viscous friction acting on a unit length of the string section

$$m^{fr} = -\mu \frac{\partial v}{\partial r} \cdot r_1 \cdot 2\pi r_1 = -\mu \frac{2\pi\omega_1 r_1^2}{r_2^2 - r_1^2}\left(r_1^2 + r_2^2\right) \tag{7}$$

When torsional vibrations of the column at a speed of ω, its total angular velocity ω_1 is

$$\omega_1 = \omega + \dot{\varphi} \tag{8}$$

And value depending on time. However, for deep drilling columns period of oscillation is large. Therefore, the process flow can be regarded as quasi-stationary, and the expression (7) can be reduced to the form

$$m^{fr} = -\mu \frac{2\pi r_1^2\left(r_1^2 + r_2^2\right)}{r_2^2 - r_1^2}(\omega + \dot{\varphi}) \tag{9}$$

and yields to

$$k = \frac{\mu 2\pi r_1^2\left(r_1^2 + r_2^2\right)}{r_2^2 - r_1^2} \tag{10}$$

Coefficient μ □ in formula (10) depends on the composition of the washing liquid and in practice varies in wide limits. As its lower limit we can take value of the viscosity of water

$\mu = 1{,}002 \cdot 10^{-3}\ Pa.\ s$, while it is equal to glycerol

$\mu = 1500 \cdot 10^{-3}\ Pa.s$.

Suppose, for example, $r_1 = 0{,}1\ м$, $r_2 = 0{,}15\ м$, then

$k \approx \frac{10^{-3} \cdot 6{,}28 \cdot 0{,}01 \cdot 0{,}0325}{0{,}0225 - 0{,}01} = \frac{10^{-5} \cdot 6{,}28 \cdot 0{,}0325}{0{,}0125} = 1{,}6328 \cdot 10^{-4}\ N \cdot s$ Let us note that although the value k is small, keeping of viscous friction in problems of torsional oscillations of the drill string may play a significant role, because, firstly, to drilling fluid μ value is much greater than for water, and secondly, this friction is implemented at great length of the column, the value of which in real terms amounts to several thousand meters.

To derive the boundary conditions for the equation (5) on the edges $z = 0$ and $z = L$, it is necessary to consider the dynamics of the bit at the point $z = 0$

and bear in mind that when $z = L$,column is clamped, which means $\varphi(L) = 0$.

If we conditionally separate bit from DS and consider it a dynamic equilibrium at $z = 0$, then the equation of the elastic torsional oscillation of the pendulum can be represented in the form of D'Alembert

$$M^{in} + M^{fr} + M^{el} = 0. \tag{11}$$

here $M^{in} = M^{in}(\ddot{\varphi})$ - is a moment of inertia forces acting on the drill bit, $M^{fr} = M^{fr}(\omega + \dot{\varphi})$ is friction torque between the bit and the destructible rock; $M^{el} = M^{el}(\varphi)$ - the moment of elastic forces, that affect the rock by screwing BC. The dots above φ mean differentiation by time t.

The value of M^{in} is calculated with formula:

$$M^{uнin} = -J \cdot \frac{\partial^2 \varphi}{\partial t^2}, \tag{12}$$

where J is a moment of inertia of rock.

The moment M^{el} is defined by the equation

$$M^{el} = GI_z \frac{\partial \varphi}{\partial z}, \tag{13}$$

where G - elastic shear modulus of the material DS, I_z - moment of inertia of DS-sectional area relative to the axis Oz.

The problem of determining the moment M^{fr} is more complex. Depending on the properties of materials of contacting bodies and the conditions of their frictional interaction different models of communication between M^{fr} and speed $\omega + \dot{\varphi}$ of their relative motion are chosen. Their formulation is performed separately.

Substituting (12) and (13) into equation (11), we rewrite it in the form of

$$J \cdot \partial^2 \varphi / \partial t^2 - M^{fr}(\omega + \partial \varphi / \partial t) + GI_z \cdot \partial \varphi / \partial z = 0 \tag{14}$$

differential equation with partial derivatives has the secondary order and a simple structure. The solution for a given ω depends on the $M^{fr}(\omega + \dot{\varphi})$ function. It can be constructed numerically for the specific initial conditions regarding $\varphi(0)$. Based on the described method developed was a set of programs to allow a model the phenomenon of self-excitation of oscillations in a wide range of characteristic parameters.

RESULTS AND DISCUSSIONS

According to the developed technique performed was computer modeling of oscillation DS by length $L = 1000\ м$ with values of the coefficient of friction $L = 1000\ ì$. The calculations were performed using the implicit finite-difference scheme for the time integration. Step of integration was accepted as equal to $\Delta t = 7{,}5 \cdot 10^{-4}\ s$.

The studies found that incorporating dissipative properties of the liquid medium in which the rotating column,

leads to narrowing of the range of values ω, which are generated during the self-oscillation. Thus, for this case it turned out that the bifurcation of cycle occurrence is realized with $\omega_p = 0{,}72\ rad/s$, and bifurcation of the loss of the cycle - at $\omega_y = 3{,}5\ rad/s$. Let us note that in the case of neglect of fluid dissipative properties, the mentioned values would be $\omega_p = 0{,}71\ rad/s$ and $\omega_y = 3{,}775\ rad/s$. Figure 5 shows a diagram of the angle φ of torsional vibrations bit of time t. It was believed that when $t < 0$, column rotates with angular velocity $\omega_p = 0{,}72\ rad/s$, but the bit was withdrawn from contact with broken rocks. Then, at $t = 0$, the bit is placed in contact with the rock, and then began the process of transition of elastic twisting columns, which replaced its fixed self-oscillation in a period of $T = 45{,}1\ s$. These fluctuations have relaxation nature, because they contain areas with almost broken outlines of $\varphi(t)$ function. Figure 6 is a graph of the angular velocity of the column $\dot{\varphi}(t)$. It is characterized by the presence of sticking zones in which $\dot{\varphi}(t)$ is approximately zero, and only short periods of time in which the sharp peaks occur.

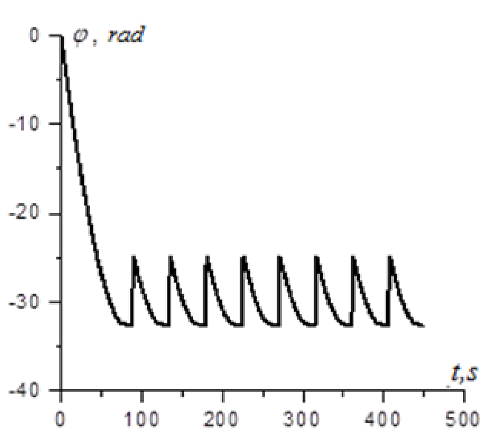

Figure 5: The Form of Relaxation Self-Oscillations at the Bit when $\omega_p = 0,72\ rad/s$

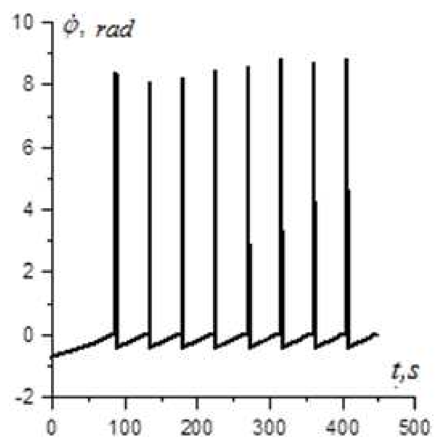

Figure 6: The Graph of Variation of the Angular Speed when $\omega_p = 0,72\ rad/s$

In a state of cycle loss when $\omega_y = 3{,}5\ rad/s$ realized are more high-frequency oscillations with a period of $T = 21{,}4\ s$ (Figure 7). Figure 8 is a graph showing changes in the angular velocity of the column $\dot{\varphi}(t)$ in a state of cycle loss.

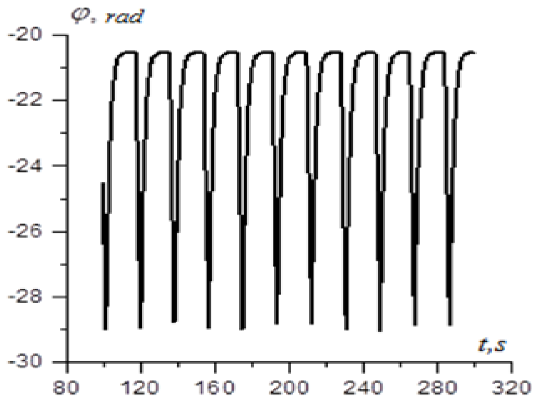

Figure 7: The Form of Oscillation Bit at $\omega_y = 3{,}5\ rad/s$

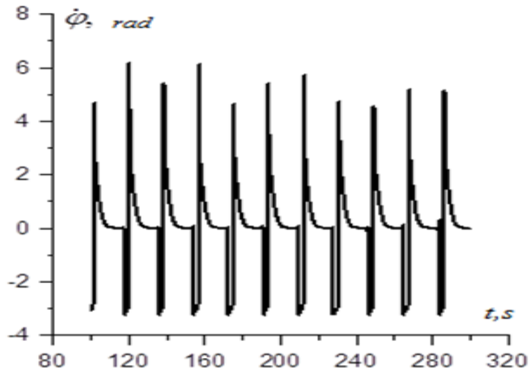

Figure 8: Graph of Changes of Angular Bit Speed at $\omega_y = 3{,}5\ rad/s$

Let us note that they also have a relaxation character and contain lots of fast and slow movements. Such regimes movements pose a serious threat to the system in real conditions, because they can be accompanied by unscrewing the bit, spalling of its diamond tools and general destruction of the column.

CONCLUSIONS

In this paper the problem about torsional vibration between a drill string in a vertical cylindrical cavity of a bore-hole and liquid medium is stated. The nonlinear differential equation with partial derivatives to describe the vibrations of the drill string bit with allowance made for viscous friction, and the algorithm for numeric integration of this equation by spatial and time variables is formulated. The mechanical interaction models of the drill string with viscous liquid are considered. The oscillation scheme of torsional auto-vibration of homogeneous drill string in the form of oscillation pendulum is stated. It is found that the properties of the liquid medium at which the column rotates, lead to a small values of angular velocity value, which are generated during the self-oscillation.

ACKNOWLEDGEMENT

The Author would like to express his sincere gratitude to the University of Philadelphia represented by the faculty of Engineering and the Deanship of scientific research and higher education for their support.

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